Multiquark cluster form factors in the relativistic harmonic oscillator model*

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Abstract: A QCD multiquark cluster system is studied in the relativistic harmonic oscillator potential model (RHOPM), and the electromagnetic form factors of the pion, proton and deuteron in the RHOPM are predicted. The calculated theoretical results are then compared with existing experimental data, finding very good agreement between the theoretical predictions and experimental data for these three target particles. We claim that this model can be applied to study QCD hadronic properties, particularly neutron properties, and to find six-quark cluster and/or nine-quark cluster probabilities in light nuclei such as helium ^{3}He and tritium ^{3}H. This is a problem of particular importance and interest in quark nuclear physics.

Key words: multiquark cluster system form factors, Relativistical Harmonic Oscillator Potential Model, quarks, QCD

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1 Introduction

Particles that interact by strong interaction are called hadrons. This general classification includes mesons and baryons. Hadrons are viewed as being composed of quarks, either as quark-antiquark pairs (mesons, with meson ware function $|M>=\frac{1}{\sqrt{3}}|q_\alpha\bar{q}_\beta>)$ where q stands for quark state and α denotes its quantum numbers, or as three quarks (baryons, with baryon ware function $|B>=\frac{1}{\sqrt{6}}\epsilon^{\alpha\beta\gamma}|q_\alpha q_\beta q_\gamma>)$. There is much more to the picture than this, however. In addition to the constituent quarks being surrounded by a cloud of gluons, they exchange particles for the strong force [1].

The quarks which determine the quantum numbers of hadrons are called valence quarks; apart from these, any hadron may contain an indefinite number of sea quarks, antiquarks and gluons, which do not influence its quantum numbers. Here, we investigate only valence quark cluster systems and do not consider the existence of sea quarks and gluons.

Our present understanding of hadrons as extended objects containing colored quarks and gluons suggests that a nucleus might not always behave as a simple collection of nucleons. Even in the loosely bound deuteron there is a small probability that the nucleons are separated by a distance less than their radius. In such a situation it seems reasonable that instead of talking of two clusters of three quarks one should speak of a single six-quark system [2]. Of course, if we were to decompose the six-quark system into clusters they could be either color singlets or octets [3]. A specific estimate of about 7% is obtained by theoretical models for the deuteron form factor [4].

In short, strongly interacting composite particles can be viewed as multiquark clusters. The deuteron is thus made up of six quarks if the proton and neutron are overlapping; in the same circumstances, ^{3}He is a system of nine quarks.

Many studies have been done using conventional methods for the form factors of strong interaction composite particles. More recently, however, hadron form factors in perturbative QCD and QCD-inspired models have been studied [5, 6]. We work in the framework of a relative harmonic oscillator potential model (RHOPM), an N-valence quark cluster system where the quarks move in a relativistic harmonic oscillator potential.

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2 Form factors of multiquark bound states

Closely following Ref. [7], we consider a system consisting of N quarks moving in the field of a relativistic harmonic oscillator potential. The corresponding wave function can be represented in the form

$$\Psi_{\rm P}^N(x_1,x_2,\cdots,x_N)=\tilde{A}\Phi_N(x_1,x_2,\cdots,x_N)U^N(\vec{P}), \quad (1)$$

where $\tilde{A}$ is the quark antisymmetrization operator (including the color degrees of freedom, which for simplicity are not written), $\Phi_N(x_1, x_2,\cdots, x_N)$ is the space-time wave function, and $U^N(\vec{P})$ is the spin wave function described below. We assume that the wave function Φ_N obeys the Klein-Gordon equation with a relativistic harmonic oscillator potential [7]

$$\left\{\sum_{i=1}^N p_i^2+\kappa^2\left[\sum_{i>j}^N\sum_{j=1}^{N-1}(x_i-x_j)^2\right]\right\}\Phi_N(x_1,x_2,\cdots,x_N)=0, \quad (2)$$

where $p_i=-\mathrm{i}\partial/\partial x_i$ is a four-momentum and κ is the oscillator parameter, x_i being the four-coordinate of the i-th quark. (We assume all quark masses are equal because of isospin invariance.) Changing to the center-of-mass coordinates X and the internal variables $r_0\cdots r_{N-1}$, and diagonalizing, one can represent Eq. (2) in the form

$$(P^2-M_{\rm p}^2)\Phi_{Nq}(r_0,r_1,\cdots,r_{N-1},P)=0, \quad (3)$$

$$M_{\rm p}^2=-2\alpha_N a_{i\mu}^+a_{i\mu}+\mathrm{const}, \quad (4)$$

$$\alpha_N=\kappa N\sqrt{N}, \quad (5)$$

where P is the total momentum of the system, $M_{\rm p}$ is the mass of the system, and $a_{i\mu}^+$ and $a_{i\mu}$ are, respectively, particle creation and annihilation operators. Under the Takabayashi condition [8], necessary for removing nonphysical oscillations along the coordinate of relative time, $p^\mu a_{i\mu}^+\Phi_{Nq}=0$, one gets the following solution

$$\Phi_{Nq}(r_0,r_1,\cdots,r_{N-1},P)$$
$$=\left(\frac{\alpha_N}{\pi N}\right)^{N-1}\exp\left(\frac{\alpha_N}{2N}K^{\mu\nu}\sum_{i=1}^{N-1}r_{i\mu}r_{i\nu}\right), \quad (6)$$

and

$$\Phi_N(x_1,x_2,\cdots,x_N)=\exp[\mathrm{i}p_\mu X_\mu]\Phi_{Nq}(r_0,r_1,\cdots,r_{n-1},P), \quad (7)$$

with $K^{\mu\nu}=g^{\mu\nu}-2p^\mu p^\nu/P^2$. From Eq. (6), then, the N-quark cluster wave function Φ_{Nq}, with the subsidiary condition formulated by Takabayasi, can be written explicitly ($n=N-1$) as:

$$\Phi_{Nq}=\left(\frac{\alpha_N}{\pi N}\right)^n\exp\left[\frac{\alpha_N}{2N}\left(g^{\mu\nu}-2\frac{p^\mu p^\nu}{M_{Nq}^2}\right)\left(\sum_{i=1}^n r_\mu^i r_\nu^i\right)\right], \quad (8)$$

where the plane wave part for the center-of-mass coordinate has been dropped. It is well known that the wave function Φ_{Nq} in Eq. (8) is characterized by the Lorentz contraction effect.

The spin wave function $U^N(\vec{P})$ can be represented in the form [9]

$$U^N(\vec{P})=B(\vec{P})U^N(0), \quad (9)$$

$$U^N(0)=\begin{pmatrix}\chi\\0\end{pmatrix},$$

$$B(\vec{P})=\exp\left[\frac{b}{2|\vec{P}|}\rho_1(\vec{P}\cdot\vec{\sigma})\right]=\exp[\rho_1 bH], \quad (10)$$

$$\rho_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

where χ is the non-relativistic spin function of the system, $H=(\vec{P}\cdot\vec{\sigma})/2|\vec{P}|$, $b=\cosh^{-1}p_0/M_{\rm p}$ and $\vec{\sigma}=\sum_{i=1}^N\vec{\sigma}^i$, with $\vec{\sigma}^i$ being the Pauli matrices of the i-th quark.

Based on Ref. [7], we write the electromagnetic action in the form

$$I_{\rm em}=\int\prod_{i=1}^N\mathrm{d}x_i\sum_k^N j_{k\mu}(x_1,x_2,\cdots,x_N)A_\mu(x_k)$$
$$\equiv\int\mathrm{d}X J_\mu^N(X)A_\mu(X), \quad (11)$$

with

$$j_{k\mu}(x_1,x_2,\cdots,x_N)$$
$$=-\mathrm{i}\Psi_{p'}^N Ne_k\left[g_{\rm E}(q^2)\frac{\overrightarrow{\partial}}{\partial x_{k\mu}}\right.$$
$$\left.+\mathrm{i}g_{\rm M}(q^2)\sigma_{\mu\nu}^k\left(\frac{\overrightarrow{\partial}}{\partial x_{k\nu}}+\frac{\overleftarrow{\partial}}{\partial x_{k\nu}}\right)\right]\Psi_{\rm p}^N. \quad (12)$$

In Eq. (12), $\Psi_{p,(p')}^N$ is the initial (final) wave function of the N-quark system as given in Eq. (1), e_k is the charge of the k-th quark, $\sigma_{\mu\nu}^k$ are the spin matrices of the k-th quark ($\sigma_{ij}^k=\varepsilon_{ijl}\sigma_l^k$, $\sigma_{i4}^k=\sigma_{4i}^k=\rho_1\sigma_i^k$), $g_{\rm E}(q^2)$ and $g_{\rm M}(q^2)$ are quark charge and magnetic form factors, and $q=p'-p$ is the four-momentum transferred to the N-quark system. Inserting the wave function Φ_N from Eq. (7) into Eqs. (11) and (12), and computing the integrals over the internal quark variables r_0, $\cdots$, r_{N-1}, one derives the matrix elements of the effective current, $J_k^N(0)$, for an N-quark system between momentum states p_μ (p'_μ) and spin component $s(s')$:

$$\langle p's'|J_\mu^N(0)|ps\rangle=\frac{I^N(q^2)}{\sqrt{2p_0p'_0}}\sum_{k=1}^N(\bar{U}_{s'}^N(p')\Gamma_{k,\mu}U_s^N(p)), \quad (13)$$

where

$$\Gamma_{k,\mu}=e_k[(p_\mu+p'_\mu)I_N(q^2)g_{\rm E}(q^2)-\mathrm{i}Ng_{\rm M}(q^2)\sigma_{\mu\nu}^{\rm k}q_\nu]. \quad (14)$$

Here the overlapping integrals over the space-time variables are the following:

$$I^N(q^2) = \frac{1}{(1+q^2/2M_{Nq}^2)^{N-1}} \times \exp\left[-\frac{N-1}{4\alpha_N}\left(\frac{q^2}{1+q^2/2M_{Nq}^2}\right)\right], \quad (15)$$

$$I_N(q^2) = \frac{1+Nq^2/2M_{Nq}^2}{1+q^2/2M_{Nq}^2}. \quad (16)$$

We now study the N-quark bound state. We can write down the form factor of an N-quark cluster system as

$$F_{Nq}(q^2) = \int \Phi_{Nq}^*(r^1,\ldots;P_F)\exp\left[-\mathrm{i}q\sum_{i=1}^{n} u_1^i r^i\right] \times \Phi_{Nq}(r^1,\cdots;P_I)\mathrm{d}^4r^1\cdots\mathrm{d}^4r^n, \quad (17)$$

where u_1^i is the first component of eigenvector u^i subject to the normalization condition

$$\sum_{i=1}^{n}|u_1^i|^2=\frac{n}{N}, \quad (18)$$

and $n=N-1$ refers to the number of relative coordinates of the constituent quarks in the multiquark system. After elementary calculation using the operator defined by $a_{r\mu}^i=\frac{1}{\sqrt{2\alpha_N}}\left(\sqrt{N}p_{r\mu}^i-\mathrm{i}\frac{\alpha_N}{\sqrt{N}}r_\mu^i\right)$, where $\alpha_N=N^{3/2}k$, and Eq. (18), Eq. (17) takes the form

$$F_{Nq}(Q^2) = \frac{1}{[1+(Q^2/2M_{Nq}^2)]^n} \times \exp\left[-\frac{n}{4\alpha_N}\frac{Q^2}{1+(Q^2/2M_{Nq}^2)}\right], \quad (19)$$

where $Q^2=-q^2$.

For the two-quark cluster of the pion ($N=2$), the form factor can be written ($n=N-1=1$) as

$$F_\pi(Q^2)=\left[1+\frac{Q^2}{2M_\pi^2}\right]^{-1}\exp\left[-\frac{1}{4\alpha_\pi}\frac{Q^2}{1+\frac{Q^2}{2M_\pi^2}}\right]. \quad (20)$$

Similarly, for the nucleon three-quark cluster ($N=3$), the form factor can be expressed as

$$F_N(Q^2)=\left[1+\frac{Q^2}{2M_N^2}\right]^{-2}\exp\left[-\frac{1}{2\alpha_N}\frac{Q^2}{1+\frac{Q^2}{2M_N^2}}\right]. \quad (21)$$

For the deuteron six-quark cluster system, once the distance between the two composite particles (proton and neutron) is less than their radius, the six-quark cluster form factor can be written

$$F_\mathrm{D}(Q^2)=\frac{1}{[1+(Q^2/2M_\mathrm{D}^2)]^5}\exp\left[-\frac{5}{4\alpha_\mathrm{D}}\frac{Q^2}{1+(Q^2/2M_\mathrm{D}^2)}\right]. \quad (22)$$

In Section 3, we compare our present theoretical results for the proton, pion and deuteron form factors with the experimental data.

3 Comparison with experimental data

Figure 1 and Fig. 2 show our present calculated electromagnetic form factors for the proton and pion respectively, compared with the corresponding experimental data [10–12]. As Figs. 1 and 2 show, there is very good agreement between the theoretical and experimental data.

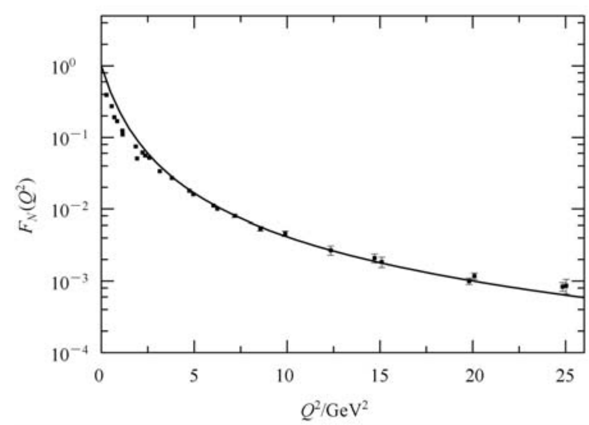

Fig. 1. Q^2-dependence of nucleon form factor $F_N(Q^2)$ and comparison with experimental data from Ref. [10].

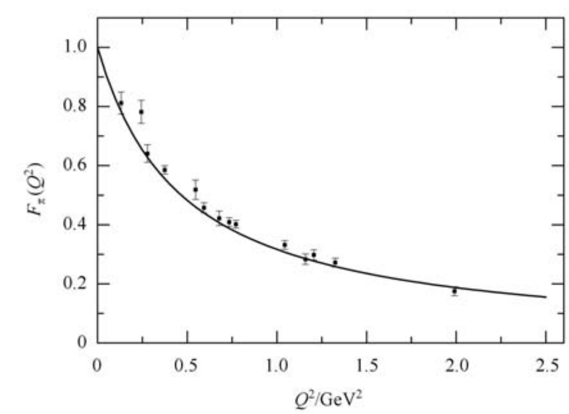

Fig. 2. Q^2-dependence of pion form factor $F_\pi(Q^2)$ and comparison with experimental data from Ref. [11].

The intriguing question of whether quark degrees of freedom play a noticeable role in understanding nuclear events has, in recent years, provoked a number of interesting works in that direction. The recent measurements of electromagnetic form factor of the deuteron at large transferred momentum as well as the indication of the existence of the cumulative effect in relativistic nuclear collisions [13] have poured new enthusiasm into attempts to treat the nucleus as a system of quarks rather than nucleons. Due to the lack of a consistent theory of quark confinement, most calculations in this field are made in the framework of the MIT quark bag model. Here, we calculate the deuteron electromagnetic form factor from the N-quark relativistic harmonic oscillator potential model.

Figure 3 shows the fit to the deuteron scalar form factor $A(Q^2)$ at high energies, where it should be possible to predict the six-quark cluster probability in the deuteron wave function. Our predicted result for the deuteron electromagnetic form factor $F_{6q}(Q^2)\sin^2(\theta)$ is approximately identical to the deuteron scalar form factor $A(Q^2)$ of the Rosenbluth separation at high energies, where $\sin^2(\theta)$ is the probability of the six-quark cluster component in the deuteron. Therefore, comparing the theoretically calculated result $F_{6q}(Q^2)\sin^2(\theta)$ with the experimental data for $A(Q^2)$, we can get the value of $\sin^2(\theta)$. This is an interesting and important issue in modern nuclear physics and hadron physics.

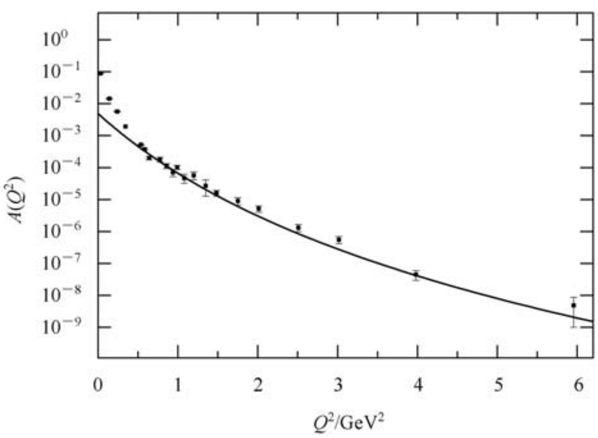

Fig. 3. Q^2-dependence of deuteron form factor $F_D(Q^2)$ and comparison with experimental data from Ref. [12]

4 Conclusions

In this paper, we studied the electromagnetic form factors of multiquark clusters in the RHOPM. Based on the belief that strongly interacting composite particles are made up of valence quarks, and assuming quarks move individually within the relativistic harmonic oscillator potential, we have calculated the electromagnetic form factor of the proton, the pion and the deuteron in the RHOPM. This model gives a fairly good simple description of these three particle structures provided only one arbitrary parameter, $g_E(q^2)=1.0$, is applied. Agreement with the corresponding experimental data is very good for all three particles.

The study of electromagnetic form factors of hadrons and nuclei has been a longstanding physical problem, on which much research work has already been published [13]. However, our theoretical investigations give not only a simple analytical expression of electromagnetic form factors of multiquark cluster systems, which is very useful for practical investigations, but also the results have pointed out a way to find six-quark and nine-quark cluster probabilities in nuclei. For example, comparing our calculated form factor for the deuteron $F_{6q}\sin^2(\theta)$ at high energies with the Rosenbluth separation form factor $A(Q^2)$ gives the deuteron six-quark cluster probability, $\sin^2(\theta)$, to be about 7%, since $F_D(Q^2)= F_{np}(Q^2)\cos^2(\theta)+F_{6q}(Q^2)\sin^2(\theta)$ and at high energies $F_{6q}(Q^2)\sin^2(\theta)=A(Q^2)$. This model can easily be extended to other mesons and baryons, as well as any system with a number of quarks larger than three, e.g. light nuclei such as ^{3}He and ^{3}H. Needless to say, finding six-quark and /or nine-quark probabilities in many-body nucleon systems is an important and interesting issue in nuclear physics which is helpful for the development of quark nuclear physics.

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